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DYNAMIC INVESTIGATIONS OF PADDLE MEMS CANTILEVERS USED IN MASS SENSING APPLICATIONS

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Abstract: *The dynamic behaviour of paddle MEMS cantilever oscillators under electrostatic actuation is investigated and presented in this paper. The scope is to estimate the influence of the geometrical dimensions and operating conditions on the frequency response of mechanical paddle cantilevers fabricated from polysilicon. Theoretical approach and finite element analysis are developed considering the multiphysics coupling between the electrical field and the mechanical structure of oscillators. The experimental tests are performed under ambient condition and in vacuum in order to characterize the effect of operating condition on the frequency response of paddle cantilevers.* **Key words:** *Mechanical oscillators, frequency response, electrostatic actuation, multiphysics coupling.*

1. INTRODUCTION

Silicone-based mass sensitive resonant sensor can be considered as silicone versions of the well-known quartz crystal microbalance (QCM). Similarly to the applications of QCMs, silicon-based mass sensitive resonators have been investigated not only as pure mass detectors but also for thin – film deposition and etching monitoring, and for humidity, chemical, and biological sensing applications. The elegance of this sensing method is that the various application fields differ only in the functional layers on the cantilever interface and the detection scheme remains common for all the different applications. The building of a mass-sensing sensor is based on oscillating cantilevers, where additional mass loading onto the cantilever interface results in a change of its resonance frequency. Compared with the other oscillating structures as microbridges or micromembranes, the microcantilevers are more sensitive to an added mass and less sensitive to temperature changes.

A mass added to a cantilever beam changes its resonance frequency. Since the mass of micromachined silicone resonator typically is of the order of 10^{-9} - 10^{-6} g, minimal detectable

mass changes of the order of 10^{-12} g and even below are feasible.

The combination of the silicone-based cantilever beams with chemically sensitive layers allows the fabrication of resonant chemical sensor.

Polymer films absorb moisture and can be used for humidity sensing. Humidity sensors are mainly used for climate control in building and process systems. A 10 μ m thick polyimide film used as sensing layer was exposed at 100% relative humidity (RH). The mass change due to the water absorption decreases the fundamental resonant frequency of cantilever. The sensor exhibits a sensibility of 2.7Hz/%RH if operate at the fundamental resonance frequency of about 16.5kHz [1].

Silicone dioxide cantilever with thickness of 1 μ m and two 8 μ m by 150 μ m cantilever beams supporting a 50 μ m by 50 μ m platform was used as the mass sensor. A mass sensitivity of 1.9 kHz/ng was measured for the device operating at the fundamental resonance frequency of 15.5kHz [2].

Different types of cantilevers such as rectangle, paddle and triangular can be used as the sensing beam. The interest of this paper is to investigate the dynamical behavior of paddle cantilevers used in mass sensing applications.

2. THEORETICAL APPROACH

The oscillators considered here are paddle microcantilevers under electrostatic actuation as sketched in figure 1.

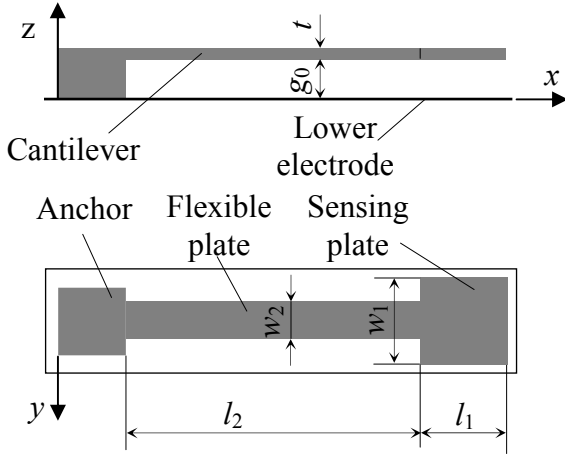


Fig. 1. Schematic of a paddle cantilever

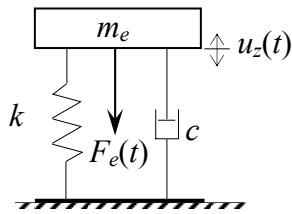


Fig. 2. A single degree of freedom model used to model the investigated electrostatic resonators

To drive the oscillator at resonance, such as in mass sensing applications, a harmonic load vibrates the cantilever. To use a paddle cantilever in mass sensing application, the eigenfrequency of beam need to be accurately determined. The eigenfrequency is equal to fundamental resonance frequency of an oscillating cantilever if the elastic properties of the cantilever remain unchanged during the mass absorption process and the beam oscillates around its initial position.

A single degree of freedom model is used to simulate the dynamic response of the oscillator as presented in figure 2. In this model the proof mass of the cantilever is modeled as a lumped mass m_e , and its stiffness is considered as a spring constant k . If a voltage $V(t)$ is applied between oscillator electrodes, the electrostatic force applied on the structure has a harmonic component with a frequency ω such as

$$F_e(t) = \frac{\epsilon A V(t)^2}{2[g_0 - u_z(t)]^2} \quad (1)$$

where ϵ is the permittivity of the free space, $A = (w_1 \times l_1) + (w_2 \times l_2)$ is the effective area of the capacitor, g_0 is the initial gap between flexible plate and substrate, and $u_z(t)$ is the displacement of the mobile plate.

The fundamental resonant frequency of an electrostatically actuated cantilever can be computed as:

$$\omega_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m_e}} \quad (2)$$

where m_e , the equivalent mass of system and k is the beam stiffness.

The stiffness of a paddle cantilever can be computed using the Castigliano's second theorem as [3]:

$$k = \frac{Et^3 w_1 w_2}{4[w_2 l_1^3 + w_1 l_2 (3l_1^2 + 3l_1 l_2 + l_2^2)]} \quad (3)$$

where w_1 , w_2 , l_1 , l_2 , t are geometrical dimensions of sample conforming to figure 1 and E is modulus of elasticity.

The equivalent mass can be calculated as [3]:

$$m_e = \frac{\rho t [\beta + \alpha + \gamma + \theta]}{140(l_1 + l_2)^6} \quad (4)$$

where ρ is the density of material, and

$$\beta = w_1 l_1^3 (33l_1^4 + 231l_1^3 l_2 + 693l_1^2 l_2^2 + 1155l_1 l_2^3 + 1155l_2^4)$$

$$\alpha = 63(10w_1 + w_2)l_1^2 l_2^5$$

$$\gamma = 7(20w_1 + 13w_2)l_1 l_2^6$$

$$\theta = 33w_2 l_2^7$$

The forced response of a mechanical oscillator determines the resonant frequency in either the presence or the absence of damping under the action of a harmonic excitation. In mechanical oscillators, the phenomenon of resonance is important, and in such situation the excitation frequency matches the resonant frequency of the system. The dynamic response of paddle cantilever oscillators subjected to a harmonic electrostatic force $F_e(t)$ with the

driving frequency ω given by an AC voltage is described by the equation of motion:

$$m \cdot \ddot{u}_z(t) + c \cdot \dot{u}_z(t) + k \cdot u_z(t) = F_e(t) \quad (5)$$

where c is the damping factor and m is the effective mass of beam.

The amplitude of oscillations under AC actuation can be written as:

$$u_z(t) = \frac{u_z}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2\xi \frac{\omega}{\omega_0}\right)^2}} \quad (6)$$

where $\xi = c/(2m\omega_0)$ is the damping ratio and u_z is the static deflection of cantilever.

The damping ratio ξ is any positive real number. For value of the damping ratio $0 \leq \xi < 1$, the system has an oscillatory response. The system damping controls the amplitude of the response when excited at resonance. When the driving frequency equals the resonant frequency $\omega = \omega_0$, the amplitude ratio reaches a maximum value. At resonance, the amplitude ratio becomes:

$$\frac{u_z(t)}{u_z} = \frac{1}{2\xi} \quad (7)$$

An important qualifier of mechanical micro-oscillators is the quality factor Q . The Q -factor is a figure of merit that takes into consideration the various energies losses in a vibrating system, which, for a harmonic oscillator, is defined as:

$$Q = 2\pi \frac{U_s}{U_d} \quad (8)$$

where U_s is the energy stored and U_d is the energy dissipated during one cycle of oscillation. At resonance, the quality factor is expressed as [4]:

$$Q_r = \frac{1}{2\xi} \quad (9)$$

and the normalized response given by equation (7) is exactly the equal to Q_r .

3. EXPERIMENTAL INVESTIGATIONS

The aims of experimental investigations are to determine the influence of the geometrical dimensions and the effect of operating conditions on the frequency response of electrostatically actuated paddle cantilevers. For experimental investigations a vibrometer analyzer is used and a white noises as an exciting signal.

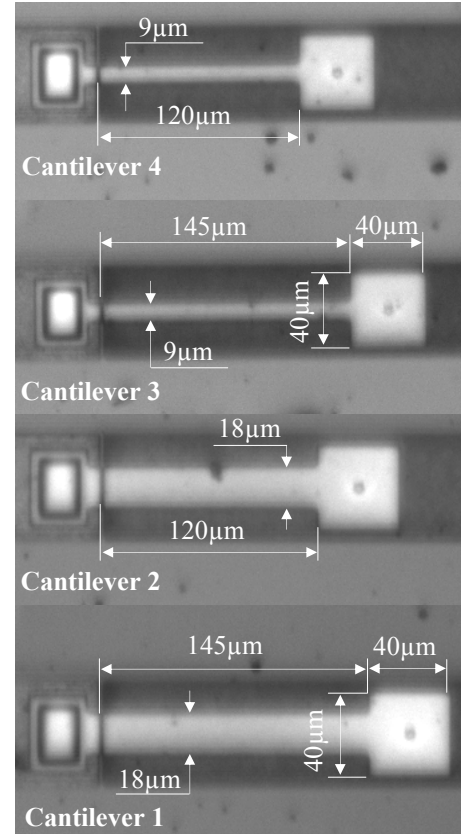


Fig. 3. Paddle cantilevers for experimental tests.

Figure 3 shows the samples for experimental investigations. These are microcantilevers fabricated from polysilicon with different positions of the sensing plate. The dimensions of sensing plate are $40\mu\text{m}$ by $40\mu\text{m}$. The thickness of beams is $1.9\mu\text{m}$ and the gap between flexible part and substrate is $2\mu\text{m}$.

During experimental tests peak amplitude of the driving signal of 3V are applied to oscillate the samples and their frequency responses are monitored. The changes in the frequency responses of cantilevers if the position of the sensing plate is moved toward to the beam anchor are experimentally determined under ambient conditions and in vacuum.

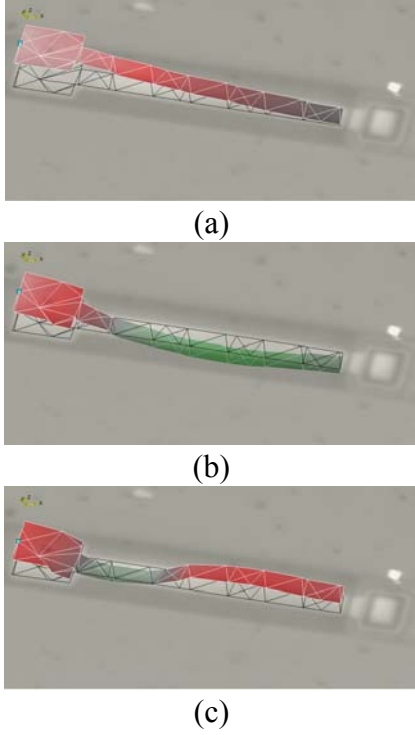


Fig. 4. Electrostatically actuated cantilever 1 under: (a) the first bending mode, (b) the second bending mode and (c) the third bending mode of oscillations

Different vibration modes of cantilevers can be visualized and analyzed. Figure 4 shows the bending modes of the cantilever 1 from figure 3 under 3V the peak amplitude of AC signal. For this cantilever, the first bending mode of oscillation corresponding to a resonant frequency of 56.3kHz is presented in figure 4a. The second mode of vibration is shown in figure 4b and it corresponds to 292.19kHz. The third bending mode is presented in figure 4c for a resonant frequency of 946.56kHz.

In order, to estimate the effect of sensing plate positions on the dynamical response of investigated beams, only the changes in the first bending mode of vibrations were monitored and analyzed. The input signal is the same for all samples: 3V peak to peak amplitude of AC current. The experiments were repeated 5 times for each sample and the average results are presented and discussed. Figures 5 and 6 present the frequency responses of the investigated cantilevers tested under ambient conditions and in vacuum. As figures show, if the length of flexible part of beam decreases from 145 μ m to 120 μ m the resonant frequencies increase about 24%. Velocities of oscillations are influenced by the operating conditions based on the damping effect.

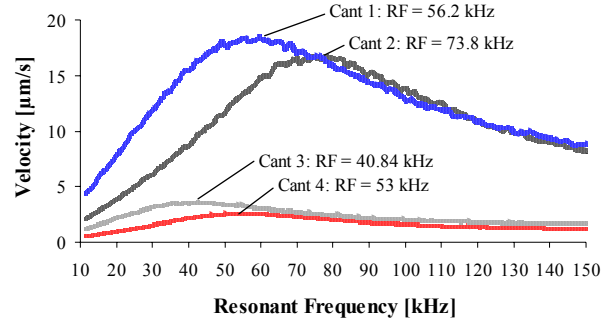


Fig. 5. Resonant frequency responses of the investigated cantilevers tested in ambient conditions

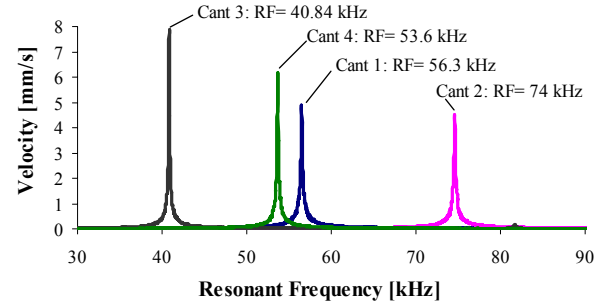


Fig. 6. Resonant frequency responses of the investigated cantilevers tested in vacuum (8×10^{-4} mbar)

Additionally, the variation of the resonant Q-factor for investigated cantilevers tested in ambient conditions are experimental monitored. Based on the frequency response experimental curves, the quality factor can be determined as:

$$Q_r = \frac{\omega}{\omega_2 - \omega_1} \quad (10)$$

where $(\omega_2 - \omega_1)$ is the frequency bandwidth corresponding to $(0.707 \times V_{\max})$.

For investigated cantilevers with a width of 18 μ m, increasing of the Q-factor from 0.82 for the cantilever 1 to 1.06 for the cantilever 2 is determined. The second set of cantilevers with a width of 9 μ m of the flexible plate, shows increasing of the Q-factor from 0.81 for cantilever 3 to 1.01 for cantilever 4. Using the values of Q-factors and based on equation (9) a shift of the damping ratio is observed as function of the sensing plate positions. The damping ratio decreases from 0.6 to 0.4 if the position of the sensing plate is moved toward to the beam anchor. As a consequence, the damping ratios is $0 \leq \zeta < 1$ which confirm that the cantilevers have oscillatory responses under the exciting signal.

4. FINITE ELEMENT ANALYSIS

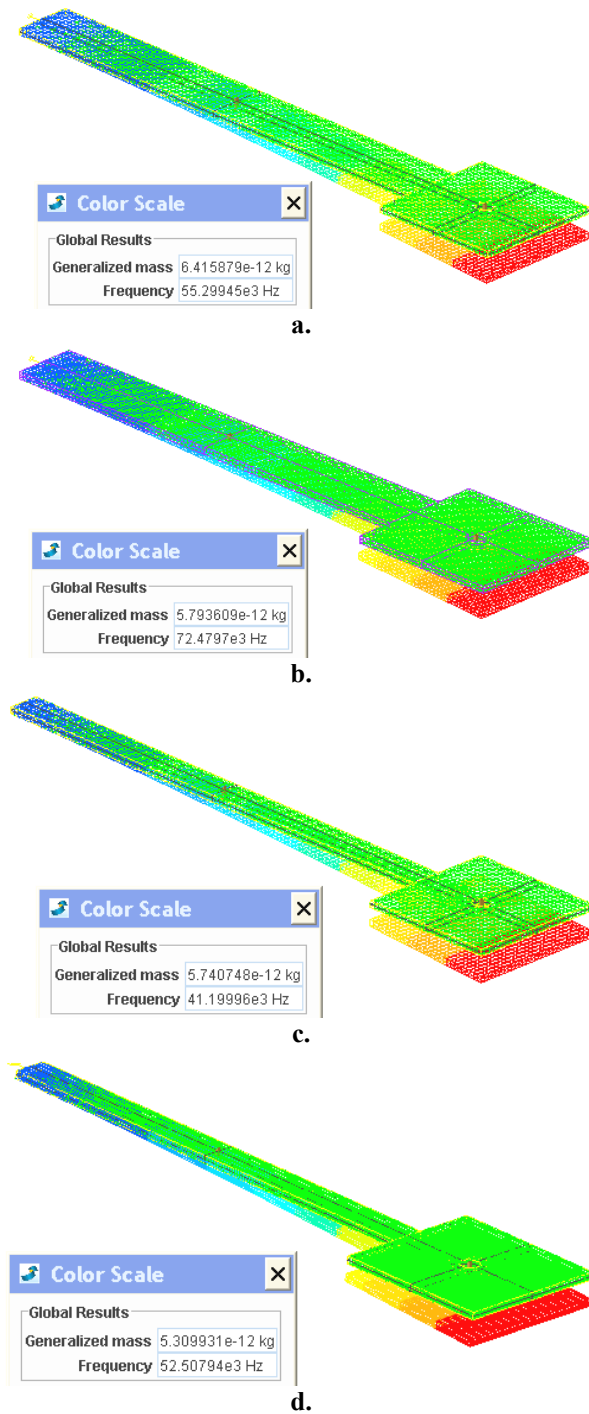


Fig. 7. Modal analysis of: (a) cantilever 1, (b) cantilever 2, (c) cantilever 3, (d) cantilever 4

To simulate the dynamic behaviour of investigated cantilevers and the change in the frequency response as function of the sensing plate position, finite element analysis is carried out using a commercial version of *Oofelie Multiphysics Simulation Software*.

The geometrical dimensions of cantilevers used in FEA are the dimensions of samples from experimental investigations.

The finite element analysis results of investigated cantilevers (Fig. 7) and their comparison with theoretical and experimental results are presented in table 1.

Table 1

Frequency responses of investigated cantilevers.

Samples	Results		
	FEA	Exp.	Th.
Cantilever 1	55,29	56,3	55,5
Cantilever 2	72,47	74	72,8
Cantilever 3	41,19	40,84	41,3
Cantilever 4	52,50	53,6	53,65

5. PADLLE CANTILEVER USED IN MASS SENSING APPLICATIONS

The use of the paddle cantilever in mass sensing applications by monitoring its change in resonant frequency response implies: a) to measure initial resonant frequency ω_0 ; b) to measure the resonant frequency ω_1 of cantilever after mass absorption; c) to estimate the added mass Δm as:

$$\Delta m = \frac{k}{4\pi^2} \left(\frac{1}{\omega_1^2} - \frac{1}{\omega_0^2} \right) \quad (11)$$

To exemplify the application of a paddle cantilever in mass sensing finite element analysis is carried out considering a thin film of 50nm thick, deposited on the sensing plate of the cantilever 3 with initial resonant frequency of 41.19 kHz (Fig.7c).

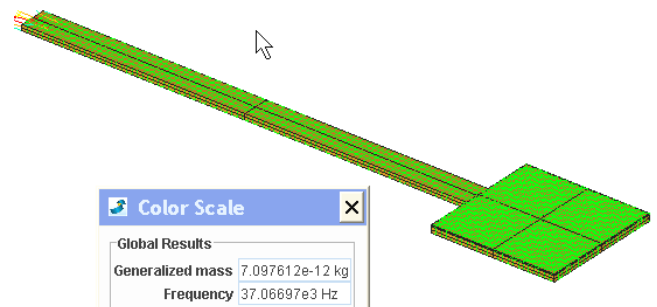


Fig. 8. Modal analysis of the investigated paddle cantilevers 3 with an added mass on the sensing plate

The resonant frequency of cantilever 3 after mass deposition is computed of 37.066 kHz (Fig.8). Using equation (11) the added mass is determined of 1.54ng. A mass sensitivity ($\Delta\omega/\Delta m$) of 2.6 kHz/ng for the cantilever 3 is estimated.

6. CONCLUSIONS

The paddle cantilevers can be used satisfactory in mass sensing application by monitoring their changes in the frequency responses. The sensitivity of cantilevers depends by the geometrical dimensions of samples. In order, to increase the sensitivity, cantilevers with high length and small width are recommended to be used. The testing conditions have influences on the minimum detectable mass. If the applications allow to be performed in vacuum, the sensitivity increases and the accuracy of measurements is improved. The experimental and finite element analysis results of frequency response are in good agreement of the investigated beams. The differences between results are influenced by the accuracy of experimental test and by the difference between material properties. In finite element analysis theoretical value of Young's modulus (taken from literature) is used.

7. REFERENCES

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INVESTIGAȚII DINAMICE LA CONSOLE MEMS DE TIP PALETA UTILIZATE IN APLICATII DE DETECTAREA MASEI

În acest articol, comportarea dinamică a consolelor MEMS de tip paleta acționate electrostatic este analizată cu scopul de a determina influențele pe care le au dimensiunile geometrice și condițiile de lucru asupra frecvenței de răspuns la cantileverele fabricate din polysilicon. Analiza teoretică și cu elemente finite este realizată considerând cuplarea multi-fizică dintre domeniul electric și cel mecanic. Experimentele sunt realizate în mediul ambiant și în vacuum cu scopul de a identifica influența pe care o are condițiile de operare asupra frecvenței de răspuns a acestor cantileverele.

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